```fortran
C       ==============================================================
C       EXAMPLE 1
C
C       REFERENCE: Example  4.1  of
C       Lee S, Chen W, Kwak B (2009) Robust design with arbitrary distributions
C     using gauss-type quadrature formula. Struct Multidisc Optim
C     39(3):227¨C243
C
C
C
C       N = 2
C       Y0 = (X(1)-4)**3 + (X(1)-3)**4 + (X(2)-5)**2 + 10
C     Y1 = X(1) + X(2) - 6.45
C       X(I) -> N(D(I),0.4**2)
C
C       NOTES:
C     Set IFLAG_AIJK = 1    :   single step PDD
C     Set IFLAG_AIJK = 0    :   Direct PDD
C
C
C******************************************************************************
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      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(200),XL(200),XU(200),G(200), OBJ(200), G1(200),
     *WK(800),IWK(200),RPRM(20),IPRM(20), XSIG_IN(200), X_IT(200,200),
     * OBJ_IT(200,200), CONS_IT(200,200),OBJ_IT_EXC(200,200),
     *  CONS_IT_EXC(200,200), XT(200), OBJ_EXC(200), G_EXC(200),
     * XLAST(200), X_IT1(200,200),
     * OBJ_IT1(200,200), CONS_IT1(200,200),OBJ_IT_EXC1(200,200),
     *  CONS_IT_EXC1(200,200), XT1(200), OBJ_EXC1(200), G_EXC1(200)

      REAL*8 XMU(20),SIG(20),NMU(20),NSIG(20)
      COMMON /SR02/ XMU,SIG, NMU, NSIG, NYQ, N

      COMMON /XR01/ N_F, N_GF, N_GC, N_EF,N_EC, N_EF1, N_EC1


      OPEN(109,FILE='ITERATION_PROCESS.TXT')
      N_F = 0
      N_GF = 0
      N_GC = 0
      N_EF = 0
      N_EC = 0
      N_EF1 = 0
      N_EC1 = 0
      X_IT = 0.0
      OBJ_IT = 0.0
      CONS_IT = 0.0
      OBJ_IT_EXC = 0.0
      CONS_IT_EXC = 0.0
      XT = 0.0
      OBJ_EXC = 0.0
      G_EXC = 0.0
      XLAST = 0.0

c     IFLAG_AIJK = 0
      IFLAG_AIJK = 1

C     DEFINE NRWK, NRIWK.
      NRWK=800
      NRIWK=200
C     ZERO RPRM AND IPRM.
      DO 10 I=1,20
        RPRM(I)=0.0
10    IPRM(I)=0
C     PROVIDE GRIDENT BY USER      --- XUCHUN REN
      IPRM(1) = 1
C     DEFINE METHOD,NDV,NCON.
C     SEQUENTIAL QUADRATIC PROGRAMMING METHOD.
      METHOD=3
C     THREE DESIGN VARIABLES.
      NDV=2
      N=NDV
C     ONE CONSTRAINT
      NCON=1
C     DEFINE BOUNDS AND INITIAL DESIGN.
      DO 20 I=1,NDV
C     INITIAL VALUES.
      X(I)=1.0
C     LOWER BOUNDS.
      XL(I)=1
C     UPPER BOUNDS
20    XU(I)=10
      X(1) = 5
      X(2) = 5
C     DEFINE IPRINT, MINMAX, INFO.
C     PRINT CONTROL.
      IPRINT=1
C     MINIMIZE
      MINMAX=-1
C     INITIALIZE INFO TO ZERO.
      INFO=0
C     OPTIMIZE.
      IITER= 0
      IITER1= 0
      IFCALL = 0
      IGFCALL = 0
      IGCCALL = 0

100   CALL DOT (INFO,METHOD,IPRINT,NDV,NCON,X,XL,XU,
     1 OBJ1,MINMAX,G1,RPRM,IPRM,WK,NRWK,IWK,NRIWK)
C     FINISHED?
      PRINT *, '*****************************************************'
      PRINT *, '*****************************************************'
      PRINT *, '*****************************************************'
      PRINT *, 'X=',X(1),X(2)
      PRINT *, '*****************************************************'
      PRINT *, '*****************************************************'
      PRINT *, '*****************************************************'
      CALL SLEEP(1)
C      PAUSE 1
      IF (IITER .GE. 1) THEN
      X_IT(IITER,:) = X
      OBJ_IT(IITER,:) = OBJ
      CONS_IT(IITER,:) = G
C     EXCTACT SOLUTION
      XT = X_IT(IITER,:)
      CALL EVAL_EXC(OBJ_EXC,XT,G_EXC)
      OBJ_IT_EXC(IITER,:) = OBJ_EXC
      CONS_IT_EXC(IITER,:) = G_EXC
      ENDIF

      IF (IITER .GE. 2) THEN
      XLAST(:) = X_IT(IITER-1,:)
      ENDIF

      IITER = IITER + 1
      IF(INFO.EQ.0) THEN
        PRINT *, 'N_F = ', N_F
        PRINT *, 'N_GF = ', N_GF
        PRINT *, 'N_GC = ', N_GC
        PRINT *, 'IITER = ', IITER
        PRINT *, 'IFCALL = ', IFCALL
        PRINT *, 'IGFCALL = ', IGFCALL
        PRINT *, 'IGCCALL = ', IGCCALL
        PRINT *, 'N_EF = ', N_EF
        PRINT *, 'N_EC = ', N_EC
        PRINT *, 'N_EF1 = ', N_EF1
        PRINT *, 'N_EC1 = ', N_EC1
        PRINT *, '---------------ITERATION PROCESS------------------'
        PRINT *, 'K      X(1)      X(2)        OBJ     OBJ_G1     OBJ_G2
     1      CONS    CONS_G1     CONS_G2 '
        WRITE(109,*) '----------------ITERATION PROCESS----------------'
        WRITE(109,*) 'X(1)      X(2)        OBJ     OBJ_G1      OBJ_G2
     1      CONS    CONS_G1     CONS_G2 '

        DO I = 1,IITER-1
            I1 = I
C            PRINT *, I, X_IT(I,1), X_IT(I,2),OBJ_IT(I1,1),OBJ_IT(I1,2)
C     1       ,OBJ_IT(I1,3), CONS_IT(I1,1), CONS_IT(I1,2), CONS_IT(I1,3)
            WRITE(109,'(I8, F20.10, F20.10, F20.10, F20.10, F20.10,
     1       F20.10, F20.10, F20.10)')
     1    I, X_IT(I,1), X_IT(I,2),OBJ_IT(I1,1),OBJ_IT(I1,2)
     2       ,OBJ_IT(I1,3), CONS_IT(I1,1), CONS_IT(I1,2), CONS_IT(I1,3)

        ENDDO
C     -----------  WRITE REDUCED ITERATION PROCESS --------
        WRITE(109,*) '----------REDUCED ITERATION PROCESS-------------'
        WRITE(109,*) 'X(1)      X(2)        OBJ     OBJ_G1      OBJ_G2
     1      CONS    CONS_G1     CONS_G2 '

        DO I = 1,IITER1
            I1 = I
            PRINT *, I, X_IT1(I,1), X_IT1(I,2),OBJ_IT1(I1,1)
     1       ,OBJ_IT1(I1,2)
     1       ,OBJ_IT1(I1,3), CONS_IT1(I1,1), CONS_IT1(I1,2),
     3        CONS_IT(I1,3)
            WRITE(109,'(I8, F20.10, F20.10, F20.10, F20.10, F20.10,
     1       F20.10, F20.10, F20.10)')
     1    I, X_IT1(I,1), X_IT1(I,2),OBJ_IT1(I1,1),OBJ_IT1(I1,2)
     2       ,OBJ_IT1(I1,3), CONS_IT1(I1,1), CONS_IT1(I1,2),
     3      CONS_IT1(I1,3)

        ENDDO

C       --  PRINT EXACT SOLUTION
        WRITE(109,*) '----------EXACT OF OBJ AND CONS--------------'
        DO I = 1,IITER-1
            WRITE(109,'(I8, F20.10, F20.10, F20.10, F20.10, F20.10,
     1       F20.10, F20.10, F20.10)')
     1    I, X_IT(I,1), X_IT(I,2),OBJ_IT_EXC(I,1),OBJ_IT_EXC(I,2)
     2       ,OBJ_IT_EXC(I,3), CONS_IT_EXC(I,1)
     3      , CONS_IT_EXC(I,2), CONS_IT_EXC(I,3)

        ENDDO


C     BEEP
        CLOSE(108)
        CLOSE(109)
        PRINT *, CHAR(7)

      STOP
C     EVALUATE OBJECTIVE AND CONSTRAINT.
      ELSE IF (INFO .EQ. 1) THEN
          IF (X(1) .NE. XLAST(1) .OR. X(2) .NE. XLAST(2)) THEN
              IF (IITER .EQ. 1) THEN
              CALL EVAL(OBJ,X,G,0,0)
              ELSEIF (IITER .GT. 1) THEN
              CALL EVAL(OBJ,X,G,0,IFLAG_AIJK)
              ENDIF
              IITER1  = IITER1 + 1
              IF (IITER1 .GE. 1) THEN
              X_IT1(IITER1,:) = X
              OBJ_IT1(IITER1,:) = OBJ
              CONS_IT1(IITER1,:) = G
C        EXCTACT SOLUTION
              XT1 = X_IT1(IITER1,:)
              CALL EVAL_EXC(OBJ_EXC1,XT1,G_EXC1)
              OBJ_IT_EXC1(IITER1,:) = OBJ_EXC1
              CONS_IT_EXC1(IITER1,:) = G_EXC1
              ENDIF

          ENDIF
          OBJ1 = OBJ(1)
          G1 = G(1)
          IFCALL = IFCALL +1


C     EVALUATE GRADIENT
      ELSE IF (INFO .EQ. 2) THEN
        IF (X(1) .NE. XLAST(1) .OR. X(2) .NE. XLAST(2)) THEN
            IITER1 = IITER1 + 1
              IF (IITER .EQ. 1) THEN
              CALL EVAL(OBJ,X,G,0,0)
              ELSEIF (IITER .GT. 1) THEN
              CALL EVAL(OBJ,X,G,0,IFLAG_AIJK)
              ENDIF


              IF (IITER1 .GE. 1) THEN
              X_IT1(IITER1,:) = X
              OBJ_IT1(IITER1,:) = OBJ
              CONS_IT1(IITER1,:) = G
C        EXCTACT SOLUTION
              XT1 = X_IT1(IITER1,:)
              CALL EVAL_EXC(OBJ_EXC1,XT1,G_EXC1)
              OBJ_IT_EXC1(IITER1,:) = OBJ_EXC1
              CONS_IT_EXC1(IITER1,:) = G_EXC1
              ENDIF

        ENDIF
        IGFCALL = IGFCALL +1
        DO I = 1,NDV
            WK(I) = OBJ(I+1)
        ENDDO
        NGT = IPRM(20)
        IF (NGT .NE. 0) THEN
C     CONSTRAINT GRADIENTS
         IGCCALL = IGCCALL +1
        DO K = 1, NGT
        DO I = 1, NDV
            WK(I+K*NDV)=G(I+1)
        ENDDO
        ENDDO
        ENDIF
      ELSE
        PRINT *, 'UNEXPECTED INFO NUMBER, INFO = ', INFO
        STOP
      ENDIF

C     GO CONTINUE WITH OPTIMIZATION.
      GO TO 100
      END




c     ----------------------------------------------------------------
      SUBROUTINE EVAL (OBJ,X,G,IMCS,IFLAG_AIJK)
CC     SUBROUTINE TO EVALUATE THE OBJECTIVE FUNCTION AND CONSTRAINTS
C     FOR THE BOX DESIGN PROBLEM.
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DOUBLE PRECISION X(*),G(200),MOMSEN_OUT(3,2),XMU_IN(200),
     1   OBJ(200), XSIG_IN(200)

        REAL*8 XMU(20),SIG(20),NMU(20),NSIG(20)
        COMMON /SR02/ XMU,SIG, NMU, NSIG, NYQ, N

        COMMON /XR01/ N_F, N_GF, N_GC, N_EF,N_EC, N_EF1, N_EC1
C        OBJ=2.0*X(2)*X(1)+2.0*X(3)*X(1)+4.0*X(2)*X(3)
C        G(1)=1.0-0.5*X(1)*X(2)*X(3)

        N_F = N_F +1
        N_GF = N_GF +1
        N_GC = N_GC +1

c        N=2
        NSAMP = 0
        IID = 0
        XSIG_IN = 0.4
        DO I = 1,N
            XMU_IN(I) = X(I)
        ENDDO
C     EVLUATE OBJECTIVE FUNCTION AND ITS SENSITIVITY
        NBAS = 4
        NBAS_SM = 1
        NGAUSS = 5
        NGAUSS_SM = 2
        NGAUSS_GQ = 5



        CALL PDD_SEN (X, XSIG_IN, NSAMP,NBAS, NBAS_SM, NGAUSS,
     1 NGAUSS_SM, IID,  NGAUSS_GQ, 1, IMCS, MOMSEN_OUT,IFLAG_AIJK)

        TMP = DSQRT((MOMSEN_OUT(1,2)-MOMSEN_OUT(1,1)**2))

        OBJ(1) = TMP/15.0D0
        OBJ(2) = 0.5*(MOMSEN_OUT(2,2) -  2.0D0* MOMSEN_OUT(1,1)
     1   *MOMSEN_OUT(2,1))/(TMP* 15.0D0)
        OBJ(3) = 0.5* (MOMSEN_OUT(3,2) -  2.0D0* MOMSEN_OUT(1,1)
     1   *MOMSEN_OUT(3,1))/(TMP * 15.0D0)

C        PAUSE 1

C     EVLUATE CONSTRAINT AND ITS SENSITIVITY
        NBAS = 1
        NBAS_SM = 1
        NGAUSS = 2
        NGAUSS_SM = 3
        NGAUSS_GQ = 2
        CALL PDD_SEN (X, XSIG_IN, NSAMP,NBAS, NBAS_SM, NGAUSS,
     1 NGAUSS_SM, IID,  NGAUSS_GQ, 2, IMCS, MOMSEN_OUT,IFLAG_AIJK)

        TMP = DSQRT((MOMSEN_OUT(1,2)-MOMSEN_OUT(1,1)**2))

        G(1) =  3.0D0*  TMP - MOMSEN_OUT(1,1)
        TMP1 =  MOMSEN_OUT(2,2) -  2.0D0* MOMSEN_OUT(1,1)
     1   *MOMSEN_OUT(2,1)
        G(2) =  3.0D0*0.5* TMP1/TMP - MOMSEN_OUT(2,1)

        TMP1 =  MOMSEN_OUT(3,2) -  2.0D0* MOMSEN_OUT(1,1)
     1   *MOMSEN_OUT(3,1)
        G(3) = 3.0D0*0.5* TMP1/TMP - MOMSEN_OUT(3,1)


        RETURN
      END




c     ----------------------------------------------------------------
      SUBROUTINE EVAL_EXC (OBJ,X,G)

        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DOUBLE PRECISION X(*),G(200),
     1   OBJ(200), XSIG(200)
      COMMON /SR02/ XMU,SIG, NMU, NSIG, NYQ, N

        XSIG = 0.4
        S = 0.4
        X1 = X(1)
        X2 = X(2)

        T_M1F = 52.0 + 43.0*S**2.0 + 3.0*S**4.0 - (60+33.0*S**2.0)*X1
     1      - 10.0*X2 + (42+6.0*S**2.0)*X1**2.0 + X2**2.0
     2      - 11.0*X1**3.0  + X1**4.0

        T0_M2F = 2704.0 + 8172.0*S**2.0 + 9651.0*S**4.0 +381.0*S**6.0
     1      + 105.0*S**8.0

        T1_M2F = (0.0 -6240.0-15994.0*S**4.0-18672.0*S**2.0
     1      - 2310.0*S**6.0)*X1
     2      + (0.0 -1040.0 - 900.0*S**2.0 - 60.0*S**4.0 )*X2

        T2_M2F = (7968.0+ 19212.0*S**2.0+9237.0*S**4.0
     1      + 420.0*S**6.0 )*X1**2.0
     1      + (240.0+ 90.0*S**2.0+ 6.0*S**4.0 )*X2**2.0
     2     + (660.0*S**2.0+1200.0 )*X1*X2

        T3_M2F = (-6184.0- 10462.0*S**2.0- 2310.0*S**4. )*X1**3.
     1       - 20.0*X2**3.0+ (-120.0 - 66.0*S**2. )*X1*X2**2.0
     2     + (-120.0*S**2.0 -840.0 )*X1**2.0*X2

        T4_M2F = (3188.0+ 3077.0*S**2.0+ 210.0*S**4. )*X1**4.
     1       + X2**4.0+ (84.0+ 12.0*S**2. )*X1**2.0*X2**2.
     2      + 220.0*X1**3.0*X2

        T5_M2F = (-1044.0- 462.0*S**2.0 )*X1**5.
     1       -22.0*X1**3.0*X2**2.0 -20.0*X1**4.0*X2

        T6_M2F = (205.0+ 28.0*S**2.0 )*X1**6.0+ 2.0*X1**4.0*X2**2.
        T7_M2F = -22.0*X1**7.
        T8_M2F = X1**8.

        T_M2F  = T0_M2F + T1_M2F + T2_M2F + T3_M2F + T4_M2F + T5_M2F +
     1      T6_M2F + T7_M2F + T8_M2F


        D1_M1F =  - (60.0+33.0*S**2.0)
     1       + 2.0*(42+6.0*S**2.0)*X1
     2      - 33.0*X1**2.0  + 4.0*X1**3.0

        D2_M1F =  - 10.0 + + 2.0*X2



        T1_D1M2F = (0.0 -6240.0-15994.0*S**4.0-18672.0*S**2.0
     1        - 2310.0*S**6.0 )


        T2_D1M2F = (7968.0+ 19212.0*S**2.0+9237.0*S**4.0
     1   + 420.0* S**6.0 )*X1*2.0
     2     + (660.0*S**2.0+1200.0 )*X1*X2

        T3_D1M2F = (-6184.0- 10462.0*S**2.0- 2310.0*S**4. )*X1**2.0*3.
     1      + (-120.0 - 66.0*S**2. )*X2**2.
     2     + (-120.0*S**2.0 -840.0 )*X1*2.0*X2

        T4_D1M2F = (3188.0+ 3077.0*S**2.0+ 210.0*S**4. )*X1**3.0*4.
     1        + (84.0+ 12.0*S**2. )*X1*2.0*X2**2.
     2     + 220.0*X1**2.0*3.0*X2

        T5_D1M2F = (-1044.0- 462.0*S**2.0 )*X1**4.0*5.
     1       -22.0*X1**2.0*2.0*X2**2.0 -20.0*X1**3.0*4.0*X2

        T6_D1M2F = (205.0+ 28.0*S**2.0 )*X1**5.0*6.0
     1                 + 2.0*X1**3.0*4.0*X2**2.
        T7_D1M2F = -22.0*X1**6.0*7.
        T8_D1M2F = X1**7.0*8.0




        D1_M2F = T1_D1M2F + T2_D1M2F + T3_D1M2F + T4_D1M2F + T5_D1M2F +
     1      T6_D1M2F + T7_D1M2F + T8_D1M2F


        T1_D2M2F = -1040.0 - 900.0*S**2.0 - 60.0*S**4.0

        T2_D2M2F = (240.0+ 90.0*S**2.0+ 6.0*S**4.0 )*X2*2.0
     2     + (660.0*S**2.0+1200.0 )*X1

        T3_D2M2F = - 20.0*X2**2.0*3.0 +(-120.0- 66.0*S**2.0 )*X1*X2*2.0
     2     + (-120.0*S**2.0 -840.0 )*X1**2

        T4_D2M2F = X2**3.0*4.0 + (84.0+ 12.0*S**2. )*X1**2.0*X2*2.0
     2     + 220.0*X1**3.0

        T5_D2M2F = -22.0*X1**3.0*X2*2.0 -20.0*X1**4.0

        T6_D2M2F =  2.0*X1**4.0*X2*2.0

        D2_M2F = T1_D2M2F + T2_D2M2F + T3_D2M2F + T4_D2M2F + T5_D2M2F
     1     + T6_D2M2F




C     ---------  G   ------
      T_M1G  = X1 + X2 -6.45
      T_M2G  = 2.0*S**2.0 + 41.6025 -12.90*(X1+X2) +X1**2.0 + X2**2.0
     1  + 2.0*X1*X2

      D1_M1G = 1
      D2_M1G = 1

      D1_M2G = -12.90 + 2.0*(X1+X2)
      D2_M2G = -12.90 + 2.0*(X1+X2)

      ST = DSQRT(T_M2F - T_M1F**2.0)
      OBJ(1) = ST/15.0D0
      OBJ(2) = 0.5*(D1_M2F - 2.0 * T_M1F * D1_M1F)/(ST*15.0D0)
      OBJ(3) = 0.5*(D2_M2F - 2.0 * T_M1F * D2_M1F)/(ST*15.0D0)

      ST = DSQRT(T_M2G - T_M1G**2.0)
      G(1) = 3.0*ST - T_M1G
      G(2) = 3.0*0.5 * (D1_M2G - 2.0 * T_M1G * D1_M1G)/ST - D1_M1G
      G(3) = 3.0*0.5 * (D2_M2G - 2.0 * T_M1G * D2_M1G)/ST - D2_M1G


        RETURN
      END
```