RELIABILITY-BASED DESIGN OPTIMIZATION OF LARGE-SCALE COMPLEX SYSTEMS

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INTRODUCTION

$$\text{Input } \mathbf{X} \in \mathbb{R}^N \rightarrow \boxed{\textbf{COMPLEX SYSTEM}} \rightarrow \text{Output } y_l(\mathbf{X}) \in \mathbb{R}$$

$$\mathbf{X} = (X_1, \cdots X_N) \in \mathbb{R}^N \sim f_{\mathbf{X}}(\mathbf{x};\mathbf{d}) \rightarrow \text{random variables}$$

$$\mathbf{d} = (d_1, \cdots d_M) \in \mathcal{D} \subseteq \mathbb{R}^M \rightarrow \text{design parameters}$$

- **Reliability-based Design Optimization (RBDO)**

$$\begin{aligned} \min_{\mathbf{d}\in\mathcal{D}\subseteq\mathbb{R}^M} \quad & c_0(\mathbf{d}) := \mathbb{E}_{\mathbf{d}}\left[y_0(\mathbf{X})\right], \\ \text{subject to} \quad & c_l(\mathbf{d}) := P_{\mathbf{d}}\left[y_l(\mathbf{X}) < 0\right] - p_l \leq 0;\ l = 1, \cdots, K, \\ & d_{k,L} \leq d_k \leq d_{k,U},\ k = 1, \cdots, M \end{aligned}$$

- **Robust Design Optimization (RDO)**

$$\begin{aligned} \min_{\mathbf{d}\in\mathcal{D}\subseteq\mathbb{R}^M} \quad & c_0(\mathbf{d}) := w_1 \mathbb{E}_{\mathbf{d}}\left[y_0(\mathbf{X})\right]/\mu_0^* + w_2\sqrt{\mathrm{var}_{\mathbf{d}}\left[y_0(\mathbf{X})\right]}/\sigma_0^*, \\ \text{subject to} \quad & c_l(\mathbf{d}) := \alpha_l\sqrt{\mathrm{var}_{\mathbf{d}}\left[y_l(\mathbf{X})\right]} - \mathbb{E}_{\mathbf{d}}\left[y_l(\mathbf{X})\right] \leq 0;\ l = 1, \cdots, K, \\ & d_{k,L} \leq d_k \leq d_{k,U},\ k = 1, \cdots, M \end{aligned}$$

INTRODUCTION

Project Goal

Create new theoretical foundations and numerical algorithms of RBDO and RDO methods for large-scale design optimization of complex engineering systems

Project Objectives

- Develop new extended polynomial dimensional decomposition (X-PDD) method for stochastic analysis of high-dimensional complex systems (Year 1)
- Integrate X-PDD and score functions for concurrent design sensitivity analysis (Year 2)
- Develop fast and efficient reliability-based and robust design optimization algorithms (Year 3)

(Project duration: April 1, 2010 - March 31, 2013)

INTRODUCTION

Accomplishments

- **Year 1**
  - Orthonormal polynomial basis and Fourier-polynomial expansions (completed)
  - Dimension-reduction integration for calculating expansion coefficients (completed)
- **Year 2**
  - Design sensitivity analysis of statistical moments (completed)
  - Global and local methods for robust design optimization (completed)
- **Year 3**
  - Design sensitivity analysis of reliability (ongoing)
  - New methods for reliability-based design optimization (ongoing)

STATISTICAL MOMENTS & SENSITIVITIES

$$\text{Input } \mathbf{X} \in \mathbb{R}^N \rightarrow \boxed{\textbf{COMPLEX SYSTEM}} \rightarrow \text{Output } y(\mathbf{X}) \in \mathcal{L}_2(\mathbb{R})$$

$$X_i \sim f_{X_i}(x_i;\mathbf{d});\ \text{indep.};\ \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\mathbf{d}) = \prod_{p=1}^{|u|}\psi_{i_p j_p}(X_{i_p};\mathbf{d})$$

- **Polynomial Dimensional Decomposition (PDD)**

$$y(\mathbf{X}) = y_\emptyset(\mathbf{d}) + \sum_{\emptyset\neq u\subseteq\{1,\cdots,N\}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}_0^{|u|} \\ j_1,\cdots,j_{|u|}\neq 0}} C_{u\mathbf{j}_{|u|}}(\mathbf{d})\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\mathbf{d})$$

- **S-variate, mth-order PDD Approximation**

$$\tilde{y}_{S,m}(\mathbf{X}) = y_\emptyset(\mathbf{d}) + \sum_{\substack{\emptyset\neq u\subseteq\{1,\cdots,N\} \\ 1\leq|u|\leq S}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}_0^{|u|},||\mathbf{j}_{|u|}||_\infty\leq m \\ j_1,\cdots,j_{|u|}\neq 0}} C_{u\mathbf{j}_{|u|}}(\mathbf{d})\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\mathbf{d})$$

$$y_\emptyset(\mathbf{d}) := \int_{\mathbb{R}^N} y(\mathbf{x})f_{\mathbf{X}}(\mathbf{x};\mathbf{d})d\mathbf{x},\ C_{u\mathbf{j}_{|u|}}(\mathbf{d}) := \int_{\mathbb{R}^N} y(\mathbf{x})\psi_{u\mathbf{j}_{|u|}}(\mathbf{x}_u;\mathbf{d})f_{\mathbf{X}}(\mathbf{x};\mathbf{d})d\mathbf{x}$$

STATISTICAL MOMENTS & SENSITIVITIES

- **Two Important Properties of Polynomial Basis**

$$\mathbb{E}_{\mathbf{d}}\left[\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\mathbf{d})\right] = 0$$

$$\mathbb{E}_{\mathbf{d}}\left[\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\mathbf{d})\psi_{v\mathbf{j}_{|v|}}(\mathbf{X}_u;\mathbf{d})\right] = \begin{cases} 1 & \text{if } u = v, \\ 0 & \text{if } u \neq v. \end{cases}$$

- **Second-Moment Statistics**

$$\mathbb{E}_{\mathbf{d}}\left[\tilde{y}_{S,m}(\mathbf{X})\right] = y_\emptyset(\mathbf{d})$$

$$\mathrm{var}_{\mathbf{d}}[\tilde{y}_{S,m}(\mathbf{X})] = \sum_{\substack{\emptyset\neq u\subseteq\{1,\cdots,N\} \\ 1\leq|u|\leq S}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}_0^{|u|},||\mathbf{j}_{|u|}||_\infty\leq m \\ j_1,\cdots,j_{|u|}\neq 0}} C^2_{u\mathbf{j}_{|u|}}(\mathbf{d})$$

STATISTICAL MOMENTS & SENSITIVITIES

- **Score Functions**

$$\begin{aligned} \frac{\partial\mathbb{E}_{\mathbf{d}}\left[y^r(\mathbf{X})\right]}{\partial d_k} &= \int_{\mathbb{R}^N} y^r(\mathbf{x})\overbrace{\frac{\partial \ln f_{X_{i_k}}(x_{i_k};\mathbf{d})}{\partial d_k}}^{:=s_k(x_{i_k};\mathbf{d})} f_{\mathbf{X}}(\mathbf{x};\mathbf{d})d\mathbf{x} \\ &:= \mathbb{E}_{\mathbf{d}}\left[y^r(\mathbf{X})s_k(X_{i_k};\mathbf{d})\right] \end{aligned}$$

$$s_k(X_{i_k};\mathbf{d}) \approx s_{k,\emptyset}(\mathbf{d}) + \sum_{j=1}^{m'} D_{i_k,j}(\mathbf{d})\psi_{i_k j}(X_{i_k};\mathbf{d})$$

- **Design Sensitivities**

$$\begin{aligned} \frac{\partial\mathbb{E}_{\mathbf{d}}\left[\tilde{y}_{S,m}(\mathbf{X})\right]}{\partial d_k} &= s_{k,\emptyset}(\mathbf{d})y_\emptyset(\mathbf{d}) + \sum_{j=1}^{m_{\min}} C_{i_k j}(\mathbf{d})D_{i_k,j}(\mathbf{d}) \\ \frac{\partial\mathbb{E}_{\mathbf{d}}\left[\tilde{y}^2_{S,m}(\mathbf{X})\right]}{\partial d_k} &= s_{k,\emptyset}(\mathbf{d})y^2_\emptyset(\mathbf{d}) + 2y_\emptyset(\mathbf{d})\sum_{j=1}^{m_{\min}} C_{i_k j}(\mathbf{d})D_{i_k,j}(\mathbf{d}) \\ &\quad + s_{k,\emptyset}(\mathbf{d})\mathrm{var}_{\mathbf{d}}[\tilde{y}_{S,m}(\mathbf{X})] + \tilde{T}_{k,m_{\min}} \end{aligned}$$

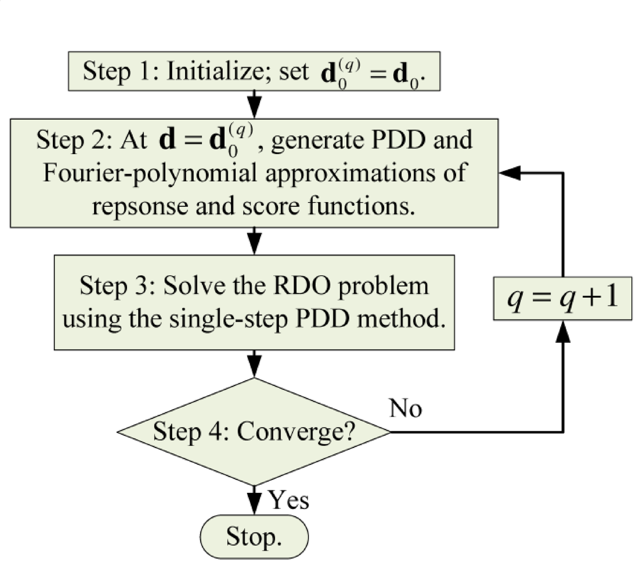
ROBUST DESIGN OPTIMIZATION

Four New Methods

- **Direct PDD (Global)**
  - Straightforward integration of PDD with gradient-based optimization algorithms
  - Re-calculation of the PDD expansion coefficients
- **Single-Step PDD (Global)**
  - Single stochastic analysis by recycling PDD coefficients
  - Premature design solutions for practical problems
- **Sequential PDD (Global)**
  - Combination of single-step and direct-PDD
  - More expensive than single-step PDD, but substantially more economical than direct PDD
- **Multipoint Single-Step PDD (Local)**
  - A succession of simpler RDO sub-problems
  - Solution of practical problems using low-order and/or low-variate PDD approximations

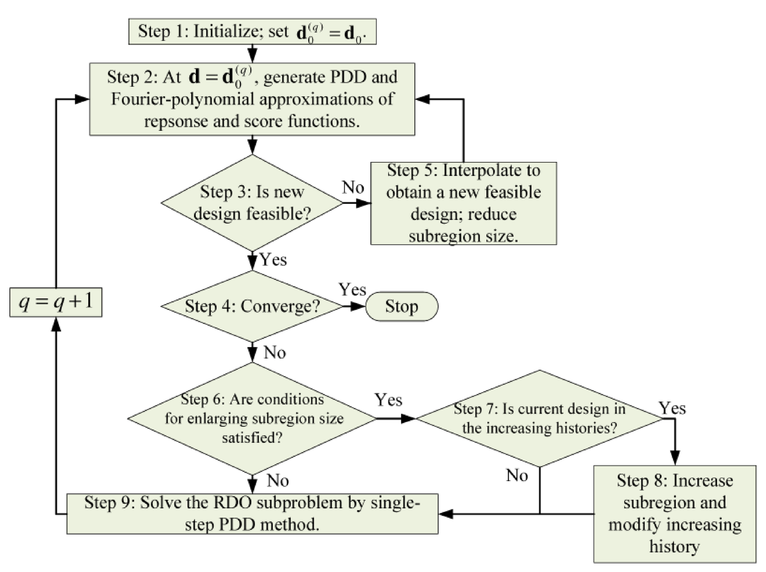
ROBUST DESIGN OPTIMIZATION

- **Sequential PDD**



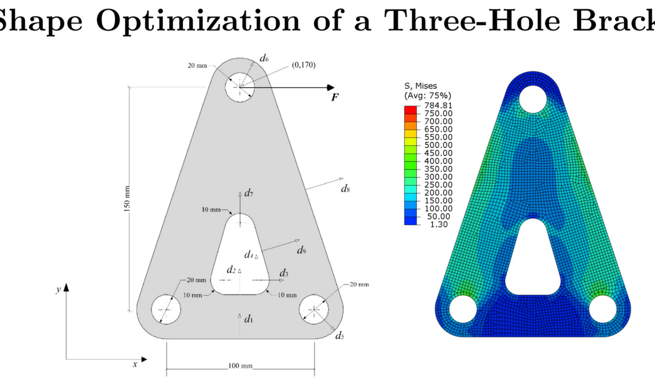
ROBUST DESIGN OPTIMIZATION

- **Multipoint Single-Step PDD**



RESULTS

- **Shape Optimization of a Three-Hole Bracket**



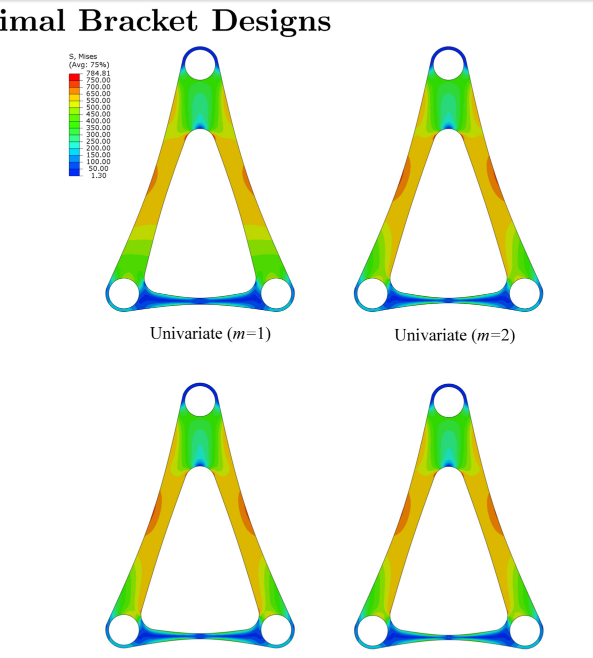
$$\begin{aligned} \min_{\mathbf{d}\in\mathcal{D}} \quad & c_0(\mathbf{d}) = 0.5\frac{\mathbb{E}_{\mathbf{d}}\left[y_0(\mathbf{X})\right]}{\mathbb{E}_{\mathbf{d}_0}\left[y_0(\mathbf{X})\right]} + 0.5\frac{\sqrt{\mathrm{var}_{\mathbf{d}}\left[y_0(\mathbf{X})\right]}}{\sqrt{\mathrm{var}_{\mathbf{d}_0}\left[y_0(\mathbf{X})\right]}}, \\ \text{subject to} \quad & c_1(\mathbf{d}) = 3\sqrt{\mathrm{var}_{\mathbf{d}}\left[y_1(\mathbf{X})\right]} - \mathbb{E}_{\mathbf{d}}\left[y_1(\mathbf{X})\right] \leq 0 \end{aligned}$$

$$y_0(\mathbf{X}) = \rho\int_{\mathcal{D}'(\mathbf{X})} d\mathcal{D}';\ y_1(\mathbf{X}) = S_y - \sigma_{e,\max}(\mathbf{X})$$

$$X_i \sim \text{truncated Gaussian variables};\ d_i = \mathbb{E}[X_i]$$

RESULTS

- **Optimal Bracket Designs**



Univariate (m=1)

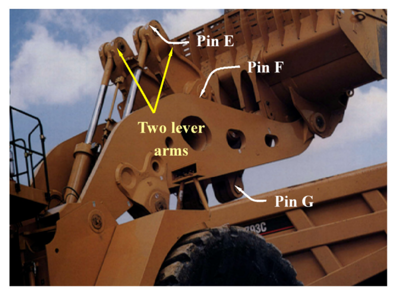
Univariate (m=2)

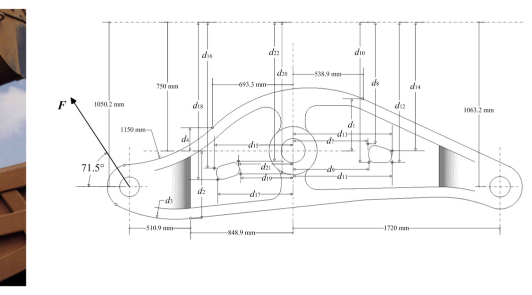
Univariate (m=3)

Bivariate (m=1)

RESULTS

- **Shape Optimization of a Lever-Arm**





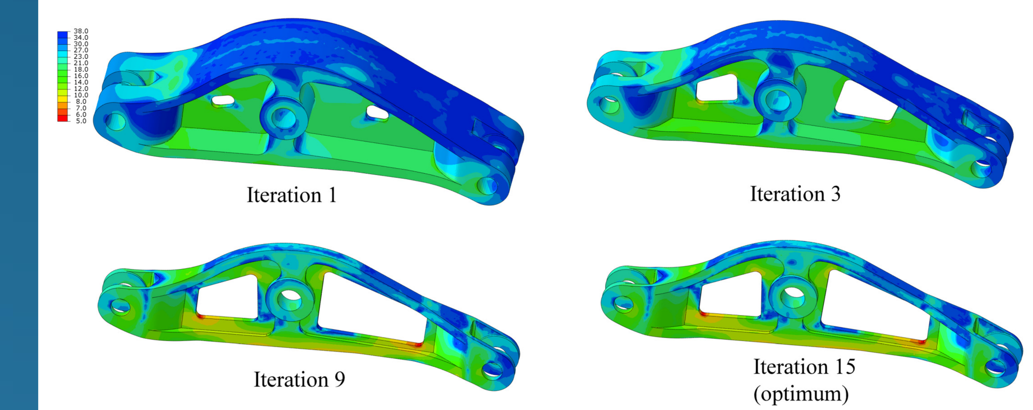
$$\begin{aligned} \min_{\mathbf{d}\in\mathcal{D}} \quad & c_0(\mathbf{d}) = 0.5\frac{\mathbb{E}_{\mathbf{d}}\left[y_0(\mathbf{X})\right]}{\mathbb{E}_{\mathbf{d}_0}\left[y_0(\mathbf{X})\right]} + 0.5\frac{\sqrt{\mathrm{var}_{\mathbf{d}}\left[y_0(\mathbf{X})\right]}}{\sqrt{\mathrm{var}_{\mathbf{d}_0}\left[y_0(\mathbf{X})\right]}}, \\ \text{subject to} \quad & c_1(\mathbf{d}) = 3\sqrt{\mathrm{var}_{\mathbf{d}}\left[y_1(\mathbf{X})\right]} - \mathbb{E}_{\mathbf{d}}\left[y_1(\mathbf{X})\right] \leq 0 \end{aligned}$$

$$y_0(\mathbf{X}) = \rho\int_{\mathcal{D}'(\mathbf{X})} d\mathcal{D}';\ y_1(\mathbf{X}) = N_{\min}(\mathbf{X}) - N_c$$

$$X_i \sim \text{truncated Gaussian variables};\ d_i = \mathbb{E}[X_i]$$

RESULTS

- **Fatigue Life Contours at Design Iterations**



Iteration 1

Iteration 3

Iteration 9

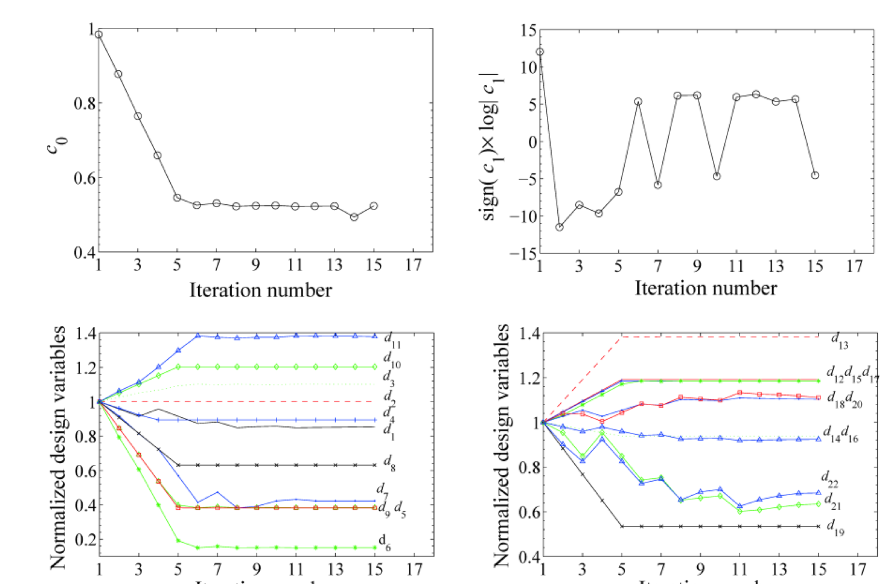
Iteration 15 (optimum)

Summary

- Optimal mass: 1263 kg (79% reduction of initial mass)
- Required 15 iterations and 675 FEA

RESULTS

- **RDO Design Histories**



BROADER IMPACT

Fundamental Aspects

- Novel optimization methods for design of complex systems subject to uncertainties
- Many stochastic problems in basic & applied sciences will be solved

Industrial Relevance

- Improved design of civil, automotive, and aerospace infrastructures
- Applications: durability, noise-vibration-harshness, creep, and crashworthiness

Knowledge Transfer

- Symposia on stochastic design optimization
- Peer-reviewed journal publications & presentations at major conferences
- Collaboration with industry (Rockwell Collins, Caterpillar)

Educational Impact

- One Ph.D. student
- Software tools in upgrading CAE & stochastic-mechanics courses
- Publication of courseware on reliability and robustness analyses and design

PUBLICATIONS

Journal

- Ren, X. and Rahman, S., "Robust Design Optimization by Polynomial Dimensional Decomposition," submitted to Structural and Multidisciplinary Optimization, 2012.
- Rahman, S. and Ren, X., "Polynomial Dimensional Decomposition for Stochastic Sensitivity Analysis of Moments," submitted to Probabilistic Engineering Mechanics, 2012.
- Rahman, S., "Uncertainty Quantification of High-Dimensional Models," submitted to SIAM Journal of Scientific Computing, 2012.
- Rahman, S., "Approximation Errors in Truncated Dimensional Decompositions," submitted to Mathematics of Computation, 2012.
- Rahman, S. and Xu. H., "Comments on High-dimensional Model Representation for Structural Reliability Analysis," International Journal for Numerical Methods in Biomedical Engineering, Vol. 27, pp. 1652-1659, 2011.
- Rahman, S., "Global Sensitivity Analysis by Polynomial Dimensional Decomposition," Reliability Engineering & System Safety, Vol. 96, No. 7, pp. 825-837, 2011.
- Rahman, S., "Statistical Moments of Polynomial Dimensional Decomposition," Journal of Engineering Mechanics, Vol. 136, No. 7, pp. 923-927, 2010.

(Others: 10 conference papers; 2 posters)